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Art. VII.—*Gravity Determinations in Australia.*

By E. F. J. LOVE, M.A., D.Sc., F.R.A.S., F. Phys. Soc. Lond.

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§ 1. Introduction.

The recent appointment, by the National Research Council of Australia, of a committee to report on the subject of a gravity survey of the continent, necessitates a critical discussion of the determinations of the gravitational acceleration which are already in existence. As regards those for Brisbane, Hobart and Perth, little can be said; each depends on a single set of observations, and, until checked, must be regarded as provisional. The work of Budik at Brisbane and Hobart is considered by Helmert[1] to be affected by a mean error of $\pm 0{\cdot}010$ cm. sec^{-2}; to that of Alessio, at Perth[2], the mean error $\pm 0{\cdot}007$ may be assigned. We therefore have, as provisional values only—

For Brisbane : $g = 979{\cdot}148 \pm 0{\cdot}010$ cm. sec^{-2}
„ Hobart : $g = 980{\cdot}441 \pm 0{\cdot}010$ „ „
„ Perth : $g = 979{\cdot}374 \pm 0{\cdot}007$ „ „

The case is different as regards the Melbourne and Sydney observatories. For each of these we have a determination by means of Kater pendulums, and several others by means of half-seconds pendulum, both of the von Sterneck and Potsdam types. Suitable averaging of these should therefore furnish definite values of g for both stations; also of their difference, which has an importance of its own, as it has already served,[3] and may possibly serve again, as a sort of "Fundamental Interval," for the calibration of gravimeters of statical type.

An error pointed out by Helmert (*l.c.*) necessitates a partial revision of the Kater pendulum reductions; this constitutes §2. §3 contains the evaluation of g for the two observatories, and §4 deals with the gravitational anomalies. The Appendices contain details which it seemed advisable to keep apart from the main paper. The notation is that in general use among geodesists. The methods of the theory of errors are used in the computing; the small quantities preceded by the sign ± are in all cases mean error.

§ 2. Revision of Results of Observations with Kater Pendulums.

Helmert (*l.c.*) has taken exception to the formula employed, both by Baracchi and myself,[4] in the reduction of our observations to the

1. Assoc. Geodes. Int., compt. rend. 13 ieme conf. gen.; II.e vol., 1901. Frequent reference is made to this paper.
2. His work at Perth and at Melbourne is about on a par; the latter is discussed in §3.
3. Threlfall and Pollock, Phil. Trans. 193 A, 1900.
4. Proc. Roy. Soc. Vict., 1893, p. 168; do., 1894, p. 8.

standard pressure of 26·in. of mercury at 32°F. His criticism is sound as regards the form, but in error as to the numerical coefficient.

The formula employed by previous workers was—

$$0.32 \left\{ \frac{B}{1+0\cdot0023(F-32)} - 26 \right\} \text{ vibration/day,}$$

where B denotes the *uncorrected* barometer reading and F that of the Fahrenheit thermometers employed. But according to the data supplied to us by General Walker,[5] the coefficient 0·32 should be replaced by 0·34. The formula used by us was, however,

$$0\cdot34 \frac{B-26}{1+0\cdot0023(F-32)} \text{ vibration/day.}$$

where B now denotes the *corrected* barometer reading; the computed vibration numbers for the pendulums were accordingly too large. Now the expression, $1+0\cdot0023(F-32)$, is really an approximation to $[+0\cdot0022(F-32)]\ [1+0\cdot0001\ (F-32)]$, the first factor being the density-temperature reduction for air (containing moisture), the second the barometer reduction. To correct the eror we must therefore add to the mean observed vibration number of each pendulum at each station the appropriate numerical value of

$$0\cdot34 \left\{ (B-26)\frac{0\cdot0001(F-32)}{1+0\cdot000.(F-32)} - 26\ \frac{0\cdot0023(F-32)}{1+0\cdot0023(F-32)} \right\}$$

For these values I obtain

Pendulum No.	Melbourne.	Sydney.
4	−0·438	−0 703
6	−0·438	−0·704
11	−0·438	−0·708

which give for the corrected vibration numbers and their differences, in place of those in our previous papers (*q.v.*)

Pendulum.	Melbourne.	Sydney.	Difference.
4	86098·83	86086·40	12·43
6	85998·99	85986·42	12·57
11	86050·62	86037·69	12·93
		Mean difference	12·64±0·15

To obtain the difference, g (Melbourne) − g (Sydney), we have, therefore, to a sufficient approximation

[g (Melbourne) − g (Sydney)]/g (Melbourne) = 2·94 10^{-4}; also g (Melbourne) = 980·0.cm sec^{-2} – – (*q.p.*) whence g (Melbourne) − g (Sydney) = 0·288 ± 0·003.cm sec^{-2}. For reasons given in Appendix 1, this figure is increased by 0·002, giving

$$g\ (\text{Melbourne}) - g\ (\text{Sydney}) = 0\cdot290.\text{cm sec}^{-2}$$

so far as the Kater pendulums[6] are concerned. The Kater pendulum

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5. General Walker's letters and "Instructions to Observers" are preserved at the Melbourne Observatory.

6. The differences obtained with half-seconds pendulums range from 0·299 to 0·313.cm sec−2.

value of g for Sydney is 979·687.cm sec^{-2} on the Potsdam system; consequently the corresponding value for Melbourne is 979·977.cm sec^{-2}. This figure replaces both the previous incorrect one, viz.: 979·969, and Borrass's semi-conjectural emendation of it, 979·993, which is given in many tables.

§ 3. Gravity at the Melbourne and Sydney Observatories on the Potsdam System.

This problem has already been discussed in part by Borrass,[7] but his discussion requires revision in view of Alessio's subsequently published work,[8] and of the results in §2 above.

Alessio's outfit was, in its main features, a replica of Hecker's;[9] their observations are characterised by much the same care and attention to detail, and each determined the flexure correction at every station, instead of trusting to its constancy as all previous experimenters had done. They differ, however, in that Hecker used five pendulums as against Alessio's four. They differ also in their manner of observing, in that Hecker used his own clock, while Alessio used clocks in regular use at the observatories, in preference to his own, arguing quite justly that a clock in steady work is less liable to systematic acceleration of rate than one recently set up;[10] on the whole, Alessio's procedure seems to be slightly the more advantageous. Comparison of their work discloses no other material advantage on either side as regards method. Nevertheless, the mean errors of their results differ, Hecker's being decidedly the smaller, especially for Sydney. Alessio's Melbourne determination is therefore assigned three-fourths, his Sydney determination one-half the weight of Hecker's.

For the relative weights, as compared with Hecker's, of other determinations in which half-seconds pendulums were used, Borrass's (*l.c.*) estimates are accepted.

As regards the Kater pendulum determinations, Helmert (*l.c.*) has assigned to that obtained at Sydney the same mean error, ±0·010, as to those of von Elblein and Budik; the corresponding mean error for the Melbourne determination—in which twice as many experimental stations are involved—would be ±0·014; but, for reasons given in Appendix 1, this is increased to ± 0 020; hence the Melbourne determination is allowed half the weight of the Sydney one.

The data for the evaluation of g for the Melbourne observatory are given in Table I.

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7. Assoc. geodes. int., compt. rend. 16 ieme conf. gen., III.e vol., p. 224. Frequent reference is made to this paper.

8. Osservazioni Gravimetrische, Genova, 1912. I owe my copy of this paper to Dr. Baldwin's kindness.

9. Hecker's masterly pendulum work is detailed in a series of monographs published by "Zentralbureau der Internationalen Erdmessung," and "Koniglich-preussisches geodatisches Institut."

10. Further details on this point are given in Appendix 2.

TABLE I.

Observer.	Type of apparatus.	g cm. sec.$^{-2}$	Weight.	Diff. from weighted mean
Baracchi-Love	Kater	979·977	0·5	−·010
v. Elblein	v. Sterneck	·991	1·	+·004
Guberth	,,	·997	1·	+·010
Hecker	Potsdam	·985	2·	−·002
Alessio	,,	·985	1·5	−·002
		Weighted mean: 979·987		
		Mean error: ±·0027		

Bernacchi's determination is omitted; Borrass also omits it. Wright's determination is, apparently, not yet published.

The data for the evaluation of g for the Sydney observatory are given in Table II.

TABLE II.

Observer.	Type of apparatus.	g cm. sec.$^{-2}$	Weight.	Diff. from weighted mean.
Smith	Kater	979·687	1·	+·007
v. Elblein	v. Sterneck	·678	1	−·002
Guberth	,,	·698	0·5	+·018
Budik	,,	·686	1·	+·006
,,	,,	·674	1·	+·006
Hecker	Potsdam	·681	2·	+·001
Alessio	,,	·675	1·	−·005
		Weighted mean: 979·680		
		Mean error: ±·0018		

Duperrey's determination (included by Borrass) is omitted, as it was not made at the observatory.

From Tables I. and II. we obtain the definite values:—

For Melbourne Observatory: g = 979·987 ± ·002$_7$ cm. sec.$^{-2}$
For Sydney ,, 979·680 ± ·001$_8$,,
For difference: g (Melb.) − g (Syd.) = 0·307 ± ·003$_2$,,

Both stations are obviously well established, the mean errors being quite small for results based on so many observations, and so large a range of methods; Sydney observatory, indeed, takes rank among the best established stations. Either would serve the purpose of a primary station for gravity survey.

§ 4 Gravitational Anomalies.

The additional figures required to obtain these anomalies are given (except those for Perth) by Borrass (*l.c.*). Taking first those for Melbourne and Sydney we obtain, in terms of the Helmert geoid of 1901, viz.:—

$$\gamma_0 = 978{\cdot}030\ (1 + 0{\cdot}005302\sin^2\phi - 0{\cdot}000007\sin^2 2\phi),$$

for Melbourne: $g_0 = 979{\cdot}995$, $g_0'' = 979{\cdot}992$, $\gamma_0 = 979{\cdot}974$,

$g_0'' - \gamma_0 = +{\cdot}018$, $g_0 - \gamma_0 = +{\cdot}021$

for Sydney: $g_0 = 979{\cdot}693$, $g_0'' = 979{\cdot}689$, $\gamma_0 = 979{\cdot}634$,

$g_0' - \gamma_0 = +{\cdot}055$, $g_0 - \gamma_0 = +{\cdot}059$

In terms of the Helmert geoid of 1915, in which the ellipticity of the equator and parallels first makes its appearance (and that with a somewhat surprisingly large coefficient for the longitude term) I find that both anomalies are much smaller. The equation of the new geoid is:—

$$\gamma_0 = 978{\cdot}052\ [1 + 0{\cdot}005285\sin^2\phi - 0{\cdot}000007\sin^2 2\phi$$
$$\pm 3 \qquad\qquad \pm 5$$
$$+ 0{\cdot}000018\cos^2\phi\cos 2(\lambda + 17)\ ]$$
$$\pm 3 \qquad\qquad \pm 4$$

where λ denotes the longitude, reckoned positive when E. of Greenwich. Hence we obtain:—

for Melbourne: $\gamma_0 = 979{\cdot}999$, $g_0'' - \gamma_0 = -{\cdot}007$, $g_0 - \gamma_0 = -{\cdot}004$,

for Sydney: $\gamma_0 = 979{\cdot}662$, $g_0'' - \gamma_0 = +{\cdot}027$, $g_0 - \gamma_0 = +{\cdot}031$.

The negative sign of these anomalies for Melbourne is noteworthy. Melbourne is, to all intent, an inland station in an extended region of low topograhic relief;[11] for it, therefore, the free-air and isostasy anomalies are not likely to differ much, and the negative sign of the former may possibly be correlated with its position in a region largely covered with relatively light rocks of late geological age, and, where of earlier age, mainly Silurian. Sydney, being a coastal station near to deep water, the correlation between its gravitational anomaly and the geological age of the neighbouring surface rocks is very likely to be masked by a large topographic effect, of which the Bouguer and free-air reductions fail to take account; a fresh reduction by the Hayford-Bowie method might clear up this point.

The anomalies, $g_0'' - \gamma_0$ and $g_0 - \gamma_0$ for Brisbane and Hobart, in terms of Helmert's 1901 geoid, are given by Borrass (*l.c.*). Those for Perth,[12] together with the anomalies for all three stations in terms of the 1915 geoid, are as follows:—

1901 geoid. Perth: $g_0 = 979{\cdot}392$, $g_0'' = 979{\cdot}387$, $\gamma_0 = 979{\cdot}477$,

$g_0'' - \gamma_0 = -{\cdot}090$, $g_0 - \gamma_0 = -{\cdot}085$*

1915 geoid, Perth: $\gamma_0 = 979{\cdot}493$, $g_0'' - \gamma_0 = -{\cdot}106$, $g_0 - \gamma_0 = -{\cdot}101$.

Brisbane: $\gamma_0 = 979{\cdot}160$, $g_0'' - \gamma_0 = -{\cdot}004$, $g_0 - \gamma_0 = .000$.

Hobart: $\gamma_0 = 980{\cdot}446$, $g_0'' - \gamma_0 = +{\cdot}007$, $g_0 - \gamma_0 = +{\cdot}013$.

The asterisked figure is also given by Alessio.

11. Port Phillip, geodetically speaking, is a shallow lake.
12. See Appenidix 4.

The anomalies for Hobart and Brisbane are reduced, those for Perth increased, by employing the new formula; the large negative values for Perth are very curious. For the reason given in §1, these anomalies must not be trusted too far; so far as they go, they favour Helmert's new formula.

Appendix 1.

In order to ascertain the proper weight to assign to the Kater pendulum determination of g for Melbourne we must investigate—

(a) The differential character of the pendulums;

(b) The relative precision of the sets of observations.

The difference between the vibration numbers of any pair of the pendulums, being small compared with the vibration numbers themselves, will be nearly the same in all four sets if differentiality is preserved. Arranging them in order of time we have—

Pendulums.	Sydney 1882 (S).	Kew 1889 (K).	Melbourne 1893 (B).	Sydney 1894 (L).
(4)-(6)	100·61	99·47	99·84	99·98
(11)-(6)	51·76	50·24	51·63	51·27

Mean: (4)-(6) = 99·98 ± ·24

Mean: (11)-(6) = 51·22 ± ·34

It is clear that the pendulums maintained their differentiality over the whole eleven years.

For the sets we obtain the following difference table:—

Pendulum.	K—S	K—B	K—L	B—S	B—L	S—L
4	66·02	58·12	70·55	7·90	12·43	4·53
6	67·18	58·49	71·06	8·67	12·57	3·90
11	65·64	57·10	70·03	8·54	12·93	4·39
Mean	66·28	57·90	70·55	8·37	12·64	4·27
Mean error	±·46	±·42	±·30	±·24	±·15	±·19

From these figures we conclude:—

1. That there is no material difference between the precision of sets S, B and L; the weakest set is K.

2. That the pendulum support was distinctly less rigid in 1893-4 than in 1882 or 1889; but slightly more rigid during my observations than during Baracchi's, by an amount apparently corresponding to an incease in the difference, g (Melbourne) $-g$ (Sydney), of about 0·002.cm sec^{-2} above that computed from the vibration numbers; hence this addition in §2. The uncertainty, however, is not entirely removed; so I have increased my estimate of the mean error from ±0·014 to the outside value ±0·020.

Appendix 2.

Alessio found, by experiments made to test the point, that his own coincidence clock was liable to accelerations of rate (sometimes positive, sometimes negative) for a few days after starting. Hecker's clock was of the same make; risky as it is to argue from the behaviour of one clock to that of another, even of the same construction, the fact is significant. Hecker also found that the mere stopping and restarting of his own clock altered its rate, on one occasion, by nearly $1\frac{1}{2}$.sec/day; so it had peculiarities of its own.

Appendix 3.

No corrections need be applied to the Sydney figures of 1882, or to the Kew figures, for the change in pressure factor from 0.32 to 0.34; the diminished pressures, under which they were obtained, were purposely chosen so as to render the pressure reductions to 26·in., at the mean temperatures of the respective observations, nearly or quite evanescent.

Appendix 4.

Mr. Curlewis informs me that the Perth observatory is built on solid sand; for reasons given in his letter, the sand appears to be of great depth. I have therefore assumed $2{\cdot}2 \pm 0{\cdot}2$ as a sufficient approximation to the density required for computing the Bouguer reduction.